Analytic Benchmarking of the 2DX eigenvalue code

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The 2DX code is a linear eigenmode solver designed for toroidal plasma configurations with an x-point topology. Together with recent upgrades to BOUT, it is part of a project to provide validation and verification capability for large-scale edge turbulence codes. By comparing linear growth rates of plasma instabilities, one avoids the problems associated with the lack of reproducibility in turbulence simulations. Comparing different turbulence codes to a single simple linear code in turn avoids the problem of assigning fault in the event of a disagreement between such codes.

Recent work on 2DX has resulted in its evolution from a fixed code for solving a specific set of equations to a generic equation language interpreter for solving nearly arbitrary systems of equations involving arbitrary numbers of fields. This greatly increases the flexibility of the code, but also raises issues of its own. Verifying correct entry of equation language instructions is simpler than verifying source code, due in part to the development of tools for visually interpreting the equation language, but remains far from trivial. Current benchmarking efforts therefore concentrate on verifying specific instruction sets, with the full capabilities of the equation language left as a feature to facilitate applications other than V&V.

Recent results in this project include a number of tests against various analytic models. These tests will also be compared to BOUT, thus cross-verifying both codes. Models used in these tests include resistive ballooning modes, resistive drift waves, ion temperature gradient (ITG) modes, and geodesic acoustic modes (GAM). By calculating the eigenmodes of these models in a simple geometry, we are able to build confidence in the 2DX code as a tool for V&V.

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Outline

- What is 2DX?
- Why create 2DX?
- Equation language
- Verification procedure
- Test cases:
  - Resistive ballooning
  - Resistive drift waves
  - Ion temperature gradient mode
  - Geodesic acoustic mode
- Conclusions

What is 2DX?

- 2DX is a linear eigensystem solver for edge instabilities in an axisymmetric toroidal geometry.
  - Differential equations converted to finite difference equations in space domain.
  - Time domain represented by eigenvalues.
  - Difference equations represented in matrix form.
  - End result is generalized eigenvalue problem:

$$Ax=\lambda Bx$$

- Equations are quasi-2D.
  - Toroidal direction represented by mode number.
  - Other directions use field line following coordinates.
- 2DX currently has the capacity to model single X-point divertor geometries.
  - Periodic and sheath boundary conditions can exist on a single grid.

Why create 2DX?

- Turbulence simulations are difficult to benchmark.
  - Nonlinear systems do not yield precisely reproducible results.
- Benchmarking results of simulations against each other are difficult to interpret.
  - Agreement is unlikely in early stages of development.
  - Different codes may have different errors, so none are actually correct.
- Linear eigenmodes can be calculated precisely and reproducibly.
  - Comparing frequencies and growth rates provides reliable benchmarking of linear terms.
- Eigenmode solvers are much simpler than full nonlinear simulations.
  - Simple codes are more reliable and easier to verify than complicated ones.
- This fills a need for verification and validation capability in edge turbulence studies.
- Allows analysis of X-point geometry using a single code for both edge and sheath.
- 2DX is also useful for fundamental edge physics analysis.
  - Unstable spectrum, extended MHD, etc.

Equation language

- Recently added feature to 2DX
- Permits solution to nearly arbitrary systems of equations
  - Equations are reduced to a series of elementary operations
  - Boundary conditions can also be reduced to these operations
  - Operations are encoded as integers
  - 2DX code uses these instructions to construct a matrix which it then solves
- Permits independent verification of equation coding
  - Equation language instructions can be translated into algebraic expressions using a Mathematica script
    - Permits intuitive comparison to written model equations
  - 2DX interpreter routines can be tested on multiple models
    - Once an instruction type is tested on one model, instructions of the same type will perform reliably on other models

- Equation language input files consist of five parts:
  - Input language: defines data file format for profile and parameter input.
  - Function language: defines functions that can be calculated from profiles and parameters.
  - Hard constant list: defines commonly used coefficients.
  - Operator language: defines how differential operators are to be represented as matrices.
  - Formula language: uses functions, constants, and operators to represent any given linear partial differential equation.

Verification procedure

- Comparison between Fortran 90 and Mathematica versions of 2DX
  - Resistive ballooning model
- Comparisons between 2DX and analytic theory
  - Resistive drift wave, ITG, GAM
- Comparisons between 2DX and BOUT
  - Resistive ballooning, resistive drift wave, ITG

Resistive ballooning test

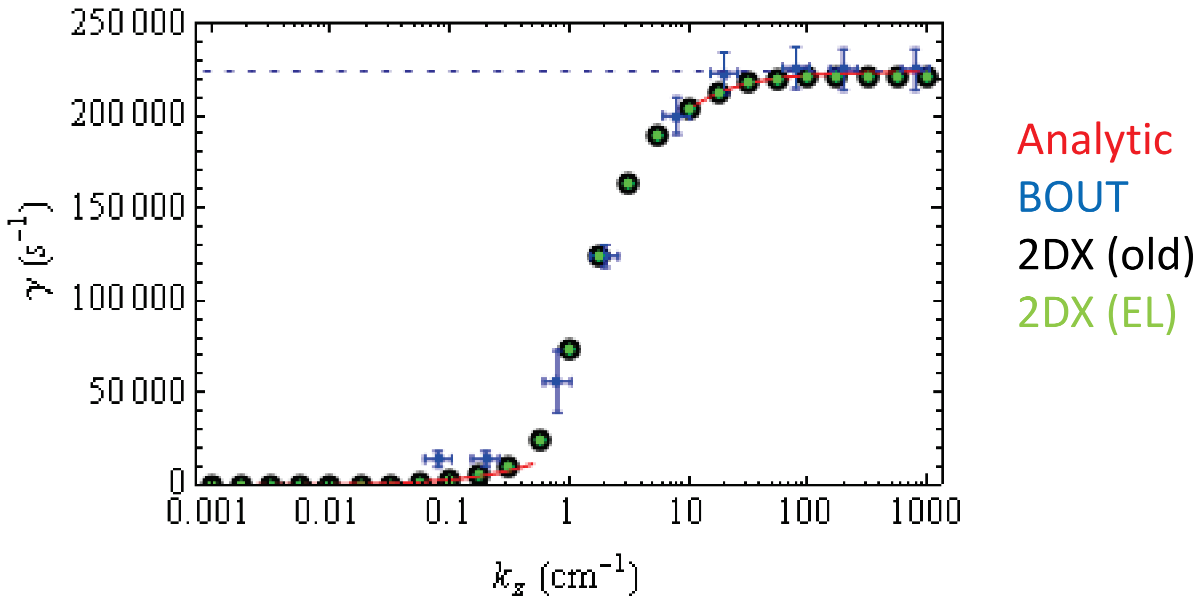
- Model equations:

$$\gamma \nabla_\perp^2 \phi = D \nabla_\parallel^2 \phi + i(A_0 + A_1 \partial_r T_0) n$$

$$\gamma n = -iF\phi$$

- This model was used in early Mathematica version of 2DX
  - Comparison verifies that newer Fortran 90/equation language version retains functionality of previous implementation
- This model has also been run on BOUT
- Analytic approximations are available for asymptotic limits

Growth rate of resistive ballooning mode



Resistive drift wave test

- Model equations:

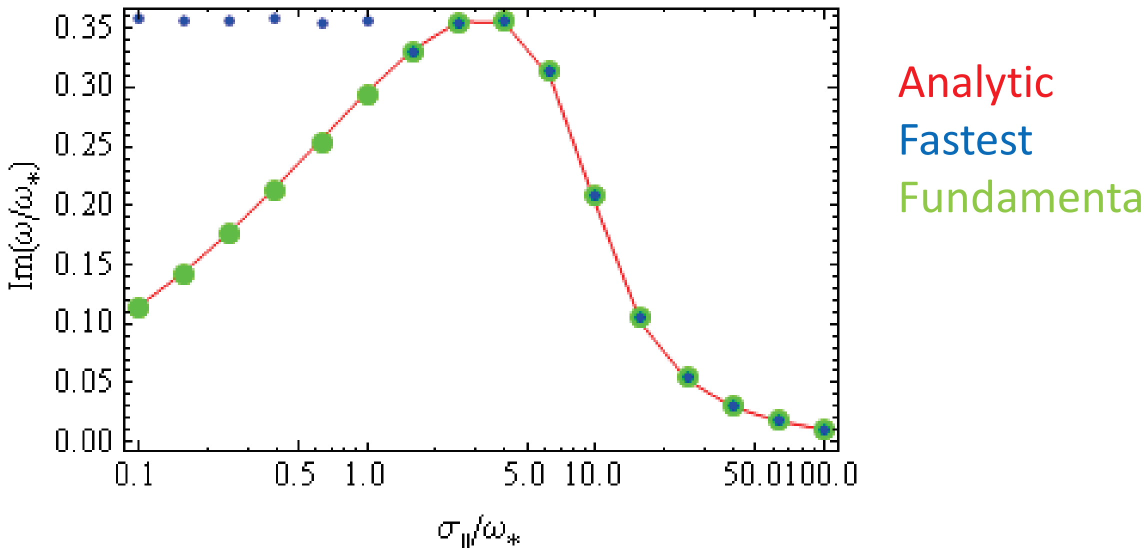
$$\gamma n = -iF\phi$$

$$\gamma \nabla_\perp^2 \phi = -\frac{B^3}{n_0} \nabla_\parallel \frac{1}{B} \nabla_\perp^2 A$$

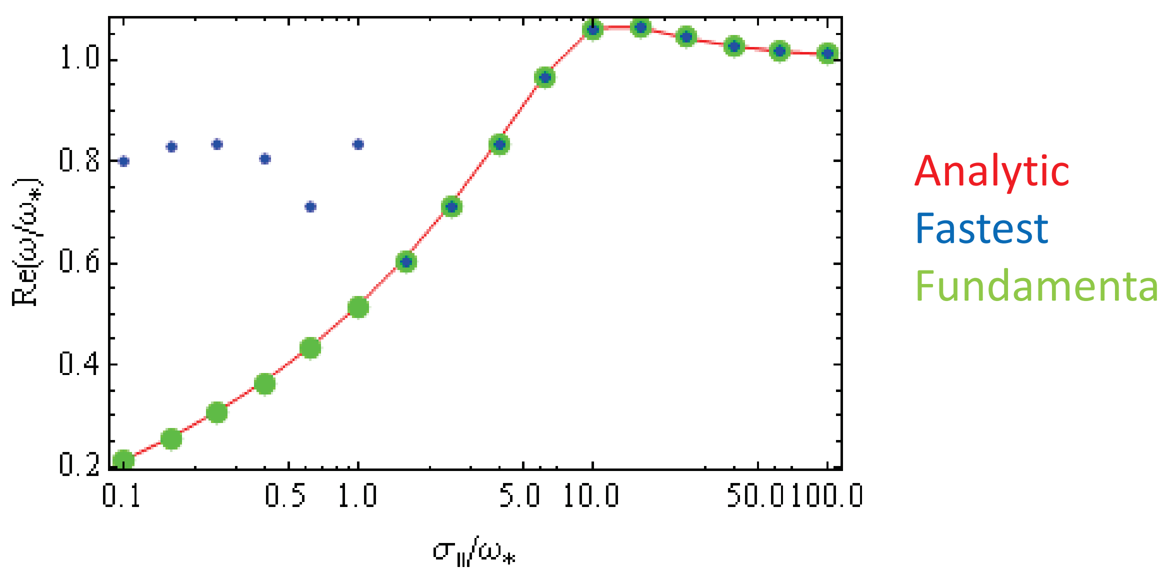
$$\gamma(1 - S\nabla_\perp^2)A = (G - iF_1\nabla_\perp^2)A - V\nabla_\parallel \phi - L\nabla_\parallel n$$

- Provides a simple 3-field model for an important instability in edge physics
- This model has also been run on BOUT
- This can be compared to analytic theory in a local (homogenous) limit

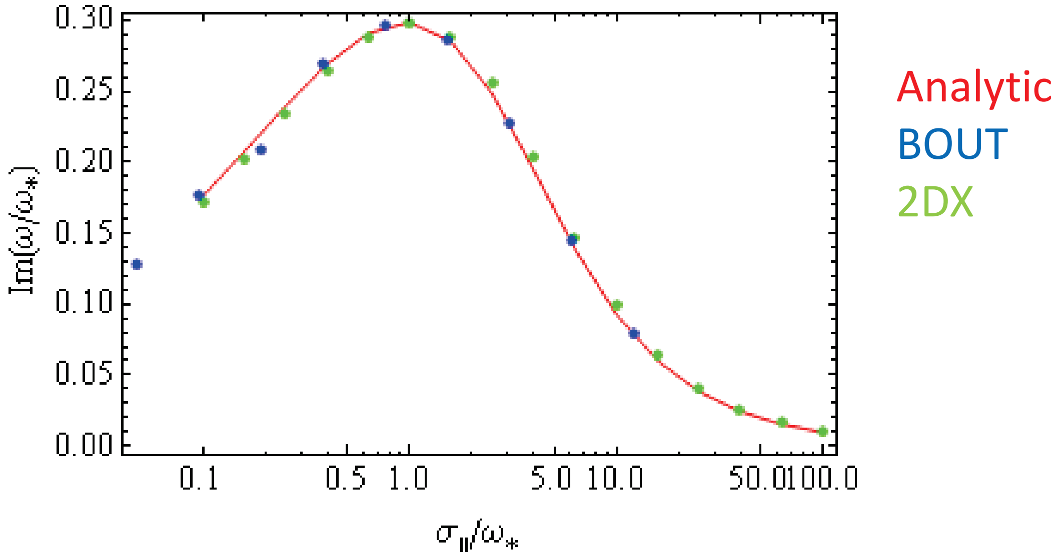
2DX test of RDW: growth rate (full EM)



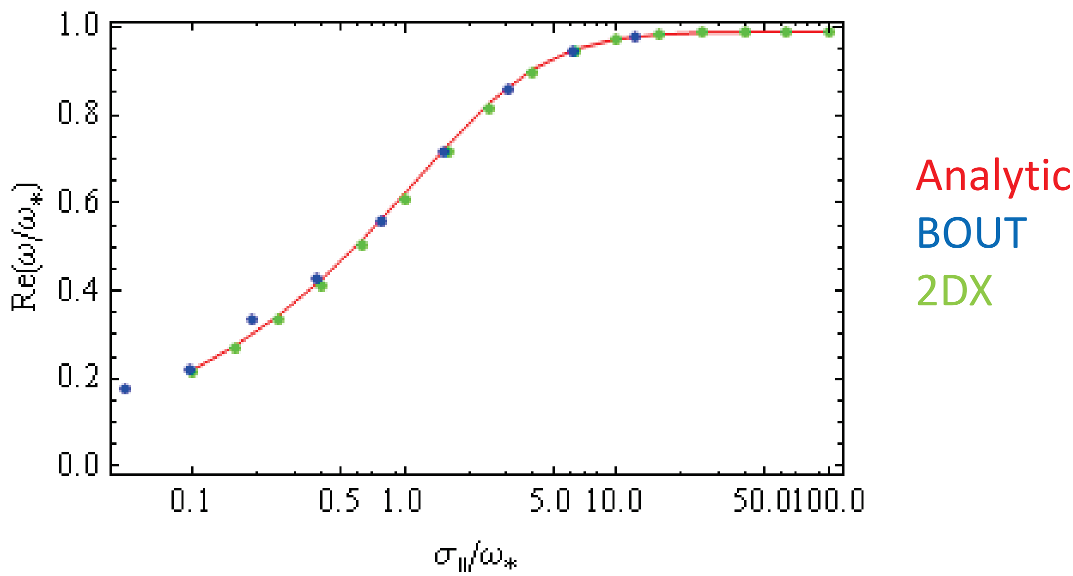
Frequency



2DX test of RDW: growth rate (ES)



Frequency



ITG test

- 3-field model equations:

$$\gamma n = -n_0 \nabla_{\parallel} \mathrm{u}$$

$$\gamma u = -\frac{T_e - T_i}{n_0} \nabla_{\parallel} \mathrm{n} - \nabla_{\parallel} \mathrm{T}$$

$$\gamma T = i k_b \frac{\partial T_i}{\partial x} \frac{n}{n_0} - \frac{2}{3} T_i \nabla_{\parallel} \mathrm{u}$$

- 5-field model equations:

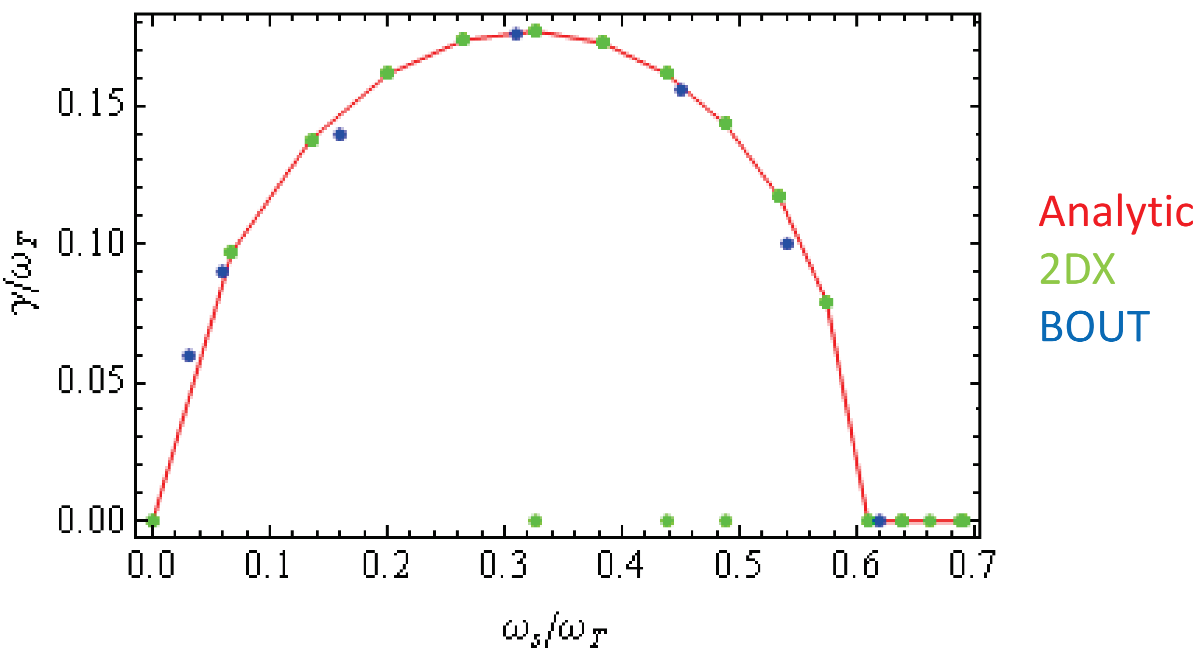
$$\gamma n = -n_0 \nabla_{\parallel} \mathrm{u} + \nabla_{\parallel} \mathrm{J} \qquad \gamma u = -\frac{T_e - T_i}{n_0} \nabla_{\parallel} \mathrm{n} - \nabla_{\parallel} \mathrm{T}$$

$$\gamma T = i k_b \frac{\partial T_i}{\partial x} \phi - \frac{2}{3} T_i \nabla_{\parallel} \mathrm{u} \qquad \gamma \nabla_{\perp}^2 \phi = \frac{1}{\mathrm{n}_0} \nabla_{\parallel} \mathrm{J}$$

$$\gamma J = -\mu n_0 \nabla_{\parallel} (\phi - \mathrm{n})$$

- First model provides simple slab-ITG test
- Second model can be simulated on BOUT
- Models should provide similar results if μ is large

Growth rate of ITG



GAM test

- Model equations:

$$\gamma \nabla_\perp^2 \phi = \frac{1}{n_0} 2\hat{b} \times \kappa \cdot \nabla(Tn) + \frac{1}{n_0} \nabla_\parallel J + \Gamma \nabla_\perp^2 \phi + \mu_{ii} \nabla_\perp^4 \phi$$

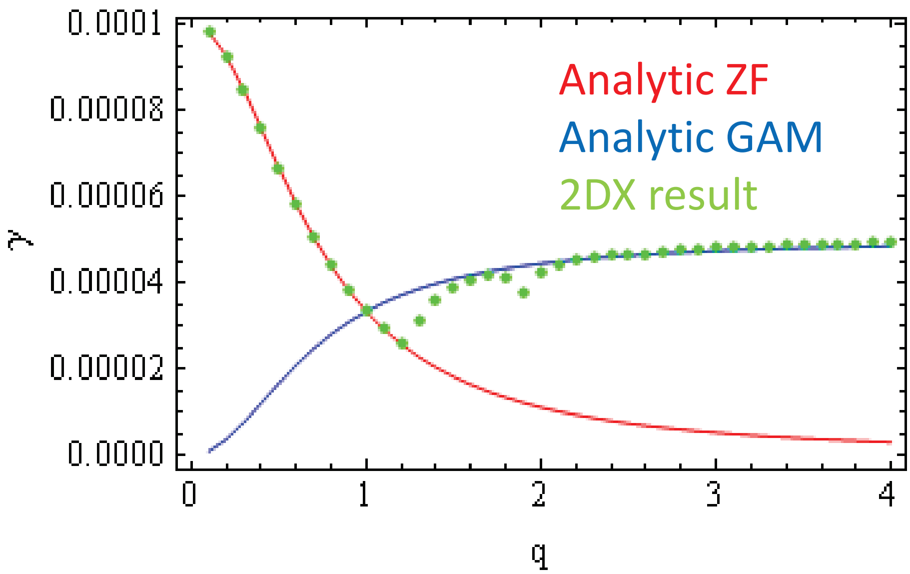
$$\gamma n = 2\hat{b} \times \kappa \cdot \nabla(Tn) - 2\hat{b} \times \kappa \cdot \nabla \phi + \nabla_\parallel J - n_0 \nabla_\parallel u$$

$$\gamma J = -\mu(n_0 \nabla_\parallel \phi - T \nabla_\parallel n) - \nu J$$

$$\gamma u = -\frac{T}{n_0} \nabla_\parallel n$$

- Provides a fluid model of geodesic acoustic mode that can also be simulated on BOUT
- Artificial drive term allows GAM to be distinguished from zonal flow
  - Mimics Reynolds stress
- Collisional damping term allows GAM to be distinguished from electron mode
- Allows 2DX to be used to study GAM

Growth rate of dominant mode in GAM model



Conclusions

- 2DX is a code for finding linear eigenmodes for fluid equations in toroidal geometry.
- 2DX has been tested for a number of models against analytic theory, BOUT, and prior versions of 2DX.
- Good agreement in all cases
- Future work:
  - Upgrade 2DX to use sparse matrices for efficient solution of large systems
  - Benchmark cases with BOUT in divertor geometry
  - Devise more reliable way to input equations, or develop fixed equation set for future runs
  - Add kinetic capabilities (2010)