Blob Stability and Transport in SOL Plasmas

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(Also Pigarov et al., this meeting, paper 2C 01)

Motivation and Introduction

- growing experimental evidence that intermittent radial transport in the SOL can be explained by coherent, propagating structures (“blobs”).

  - two-scale structure of SOL density profiles; non-diffusive in the far SOL (C-MOD, DIII-D, AUG)
  - wall recycling dominates particle fueling in C-MOD
    ⇒ importance of convective transport in far SOL
  - diagnostic imaging & probe data on C-MOD, DIII-D, NSTX, Tore Supra, MAST, and PISCES
    o turbulence ⇒ propagating coherent structures
    o responsible for large fraction of SOL transport
    o non-Gaussian statistics similar on all machines (Antar)
    o correlated with ELMs (Boedo, Endler)

(Zweben, J. Terry, Maqueda, Antar, Boedo, Rudakov, McKee, Labombard, Endler…)

- simple theoretical models of blob propagation and decay exhibit some of the qualitative features seen in the experiments.


- recent simulations extend these models to include blob dynamics and interaction with the background plasma.
GPI & BES imaging shows blobs

GPI data on NSTX
(Maqueda, Zweben, et al., 2001)

BES data on DIII-D
(Boedo, APS 2002)

BES data
5cm x 6 cm
1μs resolution
G. McKee, UW

Poloidal

Radial

Blobs are localized ⊥ B, but extended along B field lines.
Blobs move due to radial force

- species-dependent radial force $F_x$
  $(\nabla B$ (curvature), centrifugal, neutral…)
- axial B field $\Rightarrow F \times B$ drift
- sheath or plasma resistivity $\Rightarrow$ charge polarization

- velocity $u_x$ of blob depends on blob radius $a$

$$n(r) = n_b \exp\left[-\left(\frac{r^2}{2a^2}\right)\right] + n_f$$

$$n_f = 0 \quad \Rightarrow \quad u_x = \frac{q}{a^2} \quad \text{where} \quad q = \frac{L_{||}}{R}$$

- density *blobs* move out to wall ($n_b > 0$),
- density *holes* move in towards core ($n_f \neq 0$, $n_b < 0$)
Profiles depend on the blob size distribution

Ensemble average over power law distribution

\[ f(a) = a^{-p}, \quad a = \text{blob radius} \]

large blobs dominate transport for \( p = 1 \),
small blobs for \( p = 4 \)

Small blobs travel faster and penetrate farther than large blobs

Study "secondary instabilities" of blobs

- Use far SOL model \((T = \text{const})\) described by the following equations

\[
\frac{d}{dt} \nabla_{\perp}^2 \phi + \nu \nabla_{\perp}^2 \phi = \alpha \phi - \frac{\beta}{\nu} \nabla n
\]

\[
\frac{dn}{dt} = D \nabla_{\perp}^2 n - \alpha n
\]

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + \nu \cdot \nabla = \frac{\partial}{\partial t} + \mathbf{b} \times \nabla \phi \cdot \nabla
\]

\(\alpha = (2\rho_s/L_{||}) = \) sheath parameter

\(\beta = (2\rho_s/R) = \) curvature parameter

\(\nu = \) viscosity, \(D = \) diffusion

Dimensionless: \(\Omega_i \, dt \rightarrow dt, \rho_s \nabla \rightarrow \nabla,\)

\(e\Phi/T_e \rightarrow \Phi, \nu/c_s \rightarrow \nu,\) etc.

- Secondary "sheath-interchange" (SI) instability driven by internal blob pressure profile affects the radial velocity:

  - Same force drives motion and instability \((\nabla B)\)

  - SI instability \([\gamma \propto k_y^2 (\beta/\alpha L_{||})]\) breaks up blobs

  - Smaller fragments move faster, increasing transport

  (The global SOL version of this instability was studied by Nedospasov 1989, Garbet et al. 1991, Endler et al. 1995.)
Competition between stability and transport determines maximum blob size

- linear growth rate $\gamma$ for 1D "poloidally-elongated blob" ($y_b \gg x_b$)
- transport rates for blob of radius $a = x_b$
  - diffusion rate $\gamma_d = D/a^2 \ll$ other rates
  - convection rate $\gamma_c = u_x /a = q/a^3$
  - transport rate to wall $\gamma_w = u_x /\Delta x$, let $\Delta x \approx 10a$

D'Ippolito and Myra, 2003

- "essential stability" for $\gamma < \gamma_w \Rightarrow a < a_1$
- lower bound on $u_x$: $u_x > q/a_1^2$
- $a_2/u_x \Rightarrow$ maximum expected autocorrelation time
Nonlinear instability breaks up blobs

- 2D simulations $\Rightarrow$ nonlinear evolution of instability can cause blob to bifurcate

![Diagram](image)

D'Ippolito and Myra, 2003

$t/\tau_C$: (a) 0, (b) 6, (c) 9, and (d) 12; $n_f = 0.01$, $D = 0.005$, and $a_s = 7$. neglect inertia in vorticity eq.

- characteristic blob deformation before bifurcating
- also observed by G. Q. Yu et al. (2003)
Effect of background density on blobs (1)

With small background:
- blobs move faster
- blobs are unstable

Background density:
- slows and stabilizes
- changes blob shape
  + steep leading edge
  + trailing wake

D'Ippolito and Myra, 2003

The blob equilibrium, stability and transport depends on the blob height above the background.
Effect of background density on blobs (2)

- Simulations show
  - strong effect of background on radial velocity $u_x$
  - dependence of $u_x$ on blob size:

![Graph showing effect of background density on blobs](image)

D'Ippolito and Myra, 2003

- blob slows down before bifurcation and speeds up afterwards
- blob accelerates down the SOL density profile
Effect of background density on blobs (3)

- blob in vacuum \( \Rightarrow \) uniform \( u_x \)
- blob on background \( \Rightarrow \) vortex flow pattern

- sheared flow drives KH instability,
  \( \gamma \propto v_x / L_y \)

- gives steep leading edge and trailing wake

- shape \( \Rightarrow \) qualitative agreement with probe data

\[ \text{Numerical} \quad \text{Gaussian} \]
\[ n_f = 0.5 \]
\[ n - n_f \]
\[ t / \tau_c \]
Small blobs are unstable to Kelvin-Helmholtz instability

\[ a = 0.6 \text{ cm} \quad \text{(smaller blobs are more unstable to K-H)} \]

Yu, Galkin, Krasheninnikov and Pigarov, 2003

(K-H also studied by Bian et al., PoP 10, 671 (2003))
Density dips and impurity transport

Evolution of density dip with $a = 1.2$ cm, small $D = 0.01$

Yu, Galkin, Krasheninnikov and Pigarov, 2003

- density dips /holes propagate *inwards* from wall to core
- provides mechanism for enhanced impurity transport
- dips are unstable; $\gamma$ reduced by diffusion
Summary

• Blob transport provides a robust mechanism for explaining the observed intermittency and radial transport in the far SOL.

• Recent theoretical progress in understanding
  
  o blob transport and stability
  o effects of background density

• Blob motion is driven by a radial force:
  
  o grad-B due to toroidal curvature (tokamaks)
  o centrifugal force (linear machines like PISCES)
  o neutral forces, e.g. "neutral wind" in LAPD [S. Krasheninnikov, PoP 2003]

• Radial forces ⇒ blobs are unstable to secondary instabilities driven by their internal pressure and flow profiles; these instabilities affect the transport.

  o large blobs: "essential stability" criterion for sheath-interchange modes
    ⇒ maximum blob size $a_1$
    ⇒ minimum velocity $u_x > q / a_1^2$

  o small blobs: unstable to K-H instability
• 2D codes described here will be useful in interpreting and modeling experimental blob data.
  
e.g. role of vorticity, temperature gradients and velocity shear

• Radial convective flux of plasma depends on
  
  o blob size distribution
  o blob height above background density
  o ionization of neutrals (not discussed here)

• SOL convective transport has important implications for tokamaks
  
  o "main chamber recycling regime" ⇒ reduced divertor efficiency (Umansky et al., 1999; Pigarov et al., 2002)

  o may be related to the density limit on C-MOD (Greenwald, 2001; Xu, 2002; Myra et al., 2002)
Miscellaneous
Supplementary Material
(on following slides)
Blob propagation due to "neutral wind"


Outwards radial force due to imbalance between the friction of the *fast neutrals* from the core and the *slow neutrals* from the wall $\Rightarrow$ radial blob motion.

\[
\text{Force:} \quad F_{\text{Ni}} = \mu_{\text{Ni}} n \left\{ (NV)_{\text{fast}} K_{\text{fast}} + (NV)_{\text{slow}} K_{\text{slow}} \right\}
\]

\((K = \text{neutral-ion collision rate})\)

Resulting outwards velocity:

\[
v_x \propto NV_{\text{fast}} \left( \frac{K_{\text{fast}} - K_{\text{slow}}}{\Omega_i} \right) \frac{L_{\parallel}}{y_b^2}
\]

Estimate \(v_x \approx 10^5 \text{ cm/s}\) for LAPD parameters in agreement with experiment.
Reduction in blob velocity by K-H instability

2D Simulations by Yu et al (2002) show that the radial blob velocity $V_b$ decreases for small blobs due to the Kelvin-Helmholtz instability:

\[ V_b \cdot \delta^2 \]

Normalized $V_b \cdot \delta^2$ of blobs with $\delta_x = \delta_y = 0.6\sim3\text{cm}$

(Notation: $V_b = u_x$, $\delta = a$)
Stability and transport boundaries vs normalized $D$ and $\nu$

- If the inertial term in the vorticity equation is negligible, the blob radius $a$ can be scaled out of the equations; the equations are invariant under the transformation:

$$D \rightarrow a \, D, \quad a_s \rightarrow a_s/a, \quad \gamma \rightarrow \gamma \, a^3,$$

$$\varphi \rightarrow a \, \varphi, \quad n \rightarrow n$$

![Graph showing stability and transport boundaries](image-url)
Blob merger

Investigate interaction between essentially-stable blobs of different sizes:

- small fast blob merges with large slow blob
- steep leading edge and trailing wake

Parameters: $D = 0.005$, $a_s = 10$ and $n_f = 0.01$.
$t/\tau_c$: (a) 0, (b) 4.5, and (c) 9.
Effect of plasma resistivity on blob motion

- Sheath conductivity term ($\propto \alpha$) + curvature drift term ($\propto \beta$) balance in the vorticity equation to give the blob potential induced by charge polarization

$$\varphi = \frac{\beta}{\alpha n} \nabla y n = \frac{L_{\parallel}}{R_n} \nabla y n$$

- Plasma resistivity $\eta = m_e v_{ei}/n e^2$ enhances the blob potential and increases its radial velocity

$$\varphi \rightarrow \left( 1 + \frac{m_e L_{\parallel} v_{ei}}{m_i c_s} \right) \frac{L_{\parallel}}{R_n} \nabla y n$$

- Thus, plasma resistivity allows blob motion inside the separatrix and adds to sheath resistivity in SOL.

- Is plasma resistivity in SOL related to the density limit? (Xu, 2002)
Physics and scaling of K-H instability

- Kelvin-Helmholtz instability is driven by velocity shear in vorticity inertial term

\[
\frac{d}{dt} \nabla^2 \varphi = 0 \Rightarrow \gamma_{KH} \sim k_x v_x \leq \frac{v_x}{L_y}
\]

- Blob flow pattern with velocity shear requires substantial background density.

- Compare scaling of K-H growth rate with that of sheath-interchange mode

\[
\gamma_{KH} \sim \frac{v_x}{L_y} \sim \frac{1}{y_b^3}, \quad \gamma_{SI} \sim \frac{k_y^2}{L_x} \sim \frac{1}{x_b^3}
\]

- Note that \( \gamma_{KH} \) and \( \gamma_{SI} \) have opposite dependences on blob shape

  - \( \gamma_{KH} \) larger when \( x_b \gg y_b \)
  - \( \gamma_{SI} \) larger when \( y_b \gg x_b \)
Blob transport leads to non-Gaussian statistics

- analytic model of periodic blob train passing a probe:

\[ n(t) = n_0 \exp\left( \xi \left( \sin\frac{2\pi t}{\tau} - 1 \right) \right) \]

- parameter \( \xi \Rightarrow (i) \) temporal localization of the blobs, e.g. \( n(t)/n_0 \) for \( \xi = 100 \)

(iii) statistics of the density (mean ↓, skewness ↑ as \( a \uparrow \))
Characteristic Time Scales

- **Estimate blob time scales for DIII-D parameters**
  
  - blob radius: \( a = 2 \text{ cm} \)
  - distance to wall: \( w = 10 \text{ cm} \)
  - blob velocity: \( u_x = 10^5 \text{ cm/s} \)
  - connection length: \( L_\parallel = 600 \text{ cm} \)

  \[
  \Rightarrow \tau_\parallel = \frac{L_\parallel}{c_s} = 200 \mu\text{sec} \\
  \tau_w = \frac{w}{u_x} = 100 \mu\text{sec} \\
  \tau_c = \frac{a}{u_x} = 20 \mu\text{sec} 
  \]

- **Notes:**
  
  - \( \tau_w/\tau_\parallel \sim \frac{1}{2} \Rightarrow \) flattened profiles with plasma at wall
  
  - \( \tau_c \) is comparable to the experimentally-measured autocorrelation time, e.g. \( \tau = 30 \mu\text{sec} \) on NSTX