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D. A. D'Ippolito and J. R. Myra

Lodestar Research Corporation,
Boulder, Colorado, USA

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Lodestar Research Corporation

2400 Central Avenue #P-5

Boulder, CO 80301

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D. A. D'Ippolito[1,a)] and J. R. Myra[1]

[1]Lodestar Research Corporation, 2400 Central Ave. #P-5, Boulder, CO 80301
[a)]Corresponding author: dippolito@lodestar.com

Abstract. A new radio-frequency (rf) sheath boundary condition (BC) is described and applied to the problem of far field sheaths. The new BC generalizes the one presently used in rf codes to include: (1) an arbitrary magnetic field angle, (2) the full complex impedance, (3) mobile ions, (4) unmagnetized ions, and (5) the magnetic pre-sheath. For a given wave-propagation (macro) problem, root-finding is used to match the impedance of the rf wave with that of the micro-sheath problem. For a model far-field sheath problem, it is shown that the structure of the (multiple) roots with the new BC is similar to that with the capacitive BC, but the location of the resonance changes when the full impedance is used.

INTRODUCTION

An important challenge for ICRF heating is the accurate calculation of the rf sheath potential on the antenna, walls and limiters in the tokamak scrape-off-layer (SOL). One needs to know the spatial distribution and parametric dependences of the sheath potential in order to optimize the design of ICRF antennas and develop good scenarios for ICRF heating and current drive. There has been a great deal of work using an rf sheath BC [1,2] to self-consistently calculate the rf sheath potential driven by a specified rf wave. The sheath is treated as a thin vacuum layer with a capacitance, and the fields are matched across the sheath-plasma boundary by using the continuity of the normal component of the electric field, $D_n = \varepsilon\cdot\mathbf{E}\cdot s$ = constant, where **s** is the unit vector normal to the sheath. This BC [1,2] insures self-consistency between the rf fields in the plasma and sheath, and introduces new physics (e.g. sheath plasma waves (SPWs) and the SPW resonance). However, this BC has some limitations. It assumes that the ions rf response is in the "immobile" limit and that they are either strongly magnetized or the steady-state magnetic field is normal to the sheath. In these cases the capacitive impedance is much larger than the resistive part, and the magnetic pre-sheath is negligible. Since the B field is often oblique to conducting surfaces in the tokamak SOL (e.g. nearly tangent to limiter tips), this approach needs to be generalized, as discussed in Ref. [3] and in the following sections. The new sheath BC retains both the resistance and capacitance of the thin rf sheath layer, and it retains the electron and ion particle currents across the sheath, in addition to the displacement current kept previously. In this paper we will illustrate how to apply the new form of the sheath BC to the far field sheath problem discussed in Ref. [4].

SHEATH IMPEDANCE BC

Despite the nonlinear character of the rf sheath, it is useful to describe the response of the sheath as a linear lumped circuit element with a characteristic impedance Z_S at the frequency of the applied rf wave. [3] The effective impedance of a single sheath is defined by

$$V_{1\omega} = J_\omega A Z_s \equiv J_\omega z \tag{1}$$

where $V_{1\omega}$ is the change in rf voltage across the sheath, $I_\omega = J_\omega A$ is the rf current through the sheath, both at frequency ω, and $z = A\, Z_s$ is an impedance parameter. Dropping the subscript ω, the total current density across the sheath is $J = J_i + J_e + J_d$, with contributions from ions, electrons and displacement current, and A is the area of the sheath in its plane. Thus, Eq. (1) expresses a complex linear relationship between V_1 and I, which we apply to the nonlinear rf sheath to obtain an effective impedance at the driving rf frequency ω. This is done by projecting the in-phase and out-of-phase components of the current [3]. To do this, we write

$$J = \alpha V_1 + \beta \dot{V}_1 \ , \tag{2}$$

and we define $\dot{V}_1 \equiv dV_1/dt$. Multiplying both sides by V_1 and time averaging over a cycle (denoted by $\langle ... \rangle$) gives $\langle JV_1 \rangle = \alpha \langle V_1^2 \rangle$; multiplying both sides by $\dot{V}_1$ and time averaging gives $\langle J\dot{V}_1 \rangle = \beta \langle \dot{V}_1^2 \rangle$. Combining these relations with Eq. (2) gives a *voltage-current relation* for the rf sheath, which should not be confused with the nonlinear I-V characteristics of the thermal sheath.

Employing $\dot{V} = -i\omega V$ outside the time averages (the quantities inside the averages are to be computed from the nonlinear microscopic sheath model, which involve only real quantities), and using Eq. (1), we obtain the following expression [3] for the impedance parameter $z = Z_s A$:

$$\frac{1}{z} = \frac{\langle JV_1 \rangle}{\langle V_1^2 \rangle} - \frac{i\omega \langle J\dot{V}_1 \rangle}{\langle \dot{V}_1^2 \rangle} \tag{3}$$

We take Eq. (3) as the definition of the effective *rf-sheath impedance*. It can be shown that Eq. (3) reduces to the correct expressions in the resistive ($Z_s = R$) and capacitive ($Z_s = i/\omega C$) limits.

To treat the sheath self-consistently and satisfy Maxwell's equations, rf wave propagation codes need a relation between $\mathbf{E}_t$ and D_n on the plasma side of the sheath. (Here, the subscripts "n" and "t" denote the directions "normal" and "tangential" to the sheath-plasma interface.) We obtain this relation by noting that $\mathbf{E}_t$ is related to the sheath voltage V_{sh}, and D_n is related to the sheath current J. For E_t we have $\mathbf{E}_t = -\nabla_t V_{sh} = \nabla_t V_1 = \nabla_t (Jz)$ and $J = J_n = -i\omega D_n/(4\pi)$. (The sign change between V_{sh} and V_1 is explained in [3]). From these expressions we have the final result, to be employed as the new *rf sheath BC* in rf wave and antenna codes:

$$\mathbf{E}_t = \nabla_t (J_n z) = \nabla_t \left(\frac{\omega}{4\pi i} D_n z \right), \tag{4}$$

which is evaluated on the plasma side of the sheath-plasma interface. Equations (3)-(4) are very important results for future applications: they take the place of the original sheath BC, together with the Child-Langmuir Law, and extend the capacitive sheath model to include all parameter regimes for electron-poor sheaths.

THE MODEL

Now we discuss how to apply this BC for a typical wave problem. The complete sheath model consists of two parts: (i) a 1D "microscopic" sheath model on the scale of the non-neutral sheath and neutral magnetic pre-sheath (i.e. the Debye length and the ion gyroradius, respectively) and (ii) a 1D "macroscopic" sheath model on the scale of the rf wavelength or the global dimension of the SOL. For the present example, the spatial variation in both cases is assumed to be in the direction normal to the boundary. The two solutions are coupled by the generalized rf sheath BC described below, and rootfinding on the rf sheath potential is used to match the impedance of the wave to the sheath, and thus to obtain a solution to the coupled micro-macro problems.

In Ref. [3], the physics of rf sheaths near a conducting surface is studied for plasmas immersed in a magnetic field that makes an oblique angle θ with the surface. The angle θ is assumed to be large enough to insure an electron-poor sheath, and is otherwise arbitrary. In this *microscopic* model, a set of one-dimensional equations is developed that describe the dynamics of the time-dependent magnetic presheath and non-neutral Debye sheath. The model employs the continuity equation, Maxwell-Boltzmann electrons, and the three components of the ion momentum equation. The inputs are the dimensionless rf frequency, ion cyclotron frequency, driving sheath voltage, and magnetic field tilt. The ion magnetization and mobility are determined by the ion cyclotron frequency and wave frequency, respectively, relative to the ion plasma frequency. Concentrating on the ion-cyclotron range of frequencies (ICRF), the equations are solved numerically to obtain the rectified voltage, the rf voltage across the sheath and the rf current flowing through the sheath.

As an application of this model, the *rf sheath voltage-current relation* is used to obtain the *rf sheath impedance*, which in turn gives an *rf sheath boundary condition* for the electric field at the sheath-plasma interface. This BC can be used in rf wave codes to ensure proper matching and energy conservation of the waves and sheaths. In general, the impedance has both resistive and capacitive contributions, and generalizes previous capacitive sheath BC models. The resistive contribution contributes to parasitic power dissipation at the wall. As a check on the BC, one can show that the power loss reduces to that of ions accelerated out of the plasma by the rectified sheath potential in the magnetized ion limit, which is a well-known result.

The *macroscopic model* used here is a modified version of the far-field sheath model developed in Ref. [4] and used to understand some of the Alcator C-Mod plasma potential measurements [5, 6]. The model is described in detail in these papers and need not be repeated here. The physical picture is that the incoming fast wave encounters the sheath and couples to an outgoing FW and SW in order to satisfy the BC. In the earlier work (capacitive BC) the solver adjusts the sheath width until the Child-Langmuir Law is satisfied. In the present work, the incoming wave encounters a sheath with complex impedance and adjusts the driving rf voltage at the sheath entrance until the BC from the microscopic model is simultaneously satisfied in the macroscopic model. The solution determines the ratio of the three wave amplitudes and the complex sheath impedance

It is not really practical to directly solve the microscopic sheath model in the rootfinder for the macroscopic calculation. Instead here we used an ad hoc analytic model which retains the displacement and electron currents across the sheaths but neglects the ion current. This model is a stand-in for the full impedance model and is accurate to within a factor of 2 or so. We also use a capacitive model in which only the displacement current is retained in z, which is compared with the old sheath BC. In future work, we will develop analytic and numerical fits for the impedance computed by the full microscopic model including the ion current.

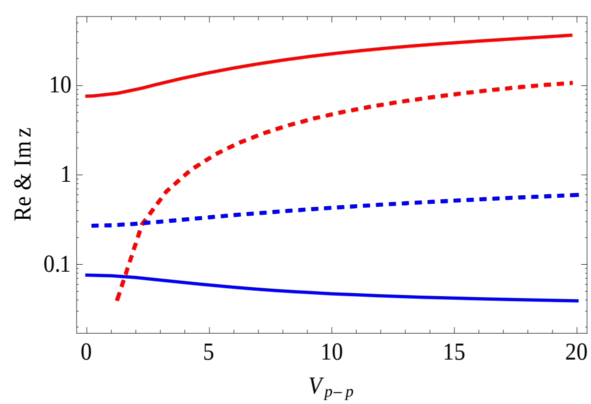

FIGURE 1. Real (solid) and imaginary (dashed) parts of z = Z A vs the peak-to-peak driving voltage for two contrasting micro-sheath solutions (described in text).

NUMERICAL RESULTS

An example of two contrasting microscopic sheath solutions is shown in Fig. 1 where the real (solid) and imaginary (dashed) parts of z are plotted vs the peak-peak driving voltage (all quantities dimensionless: $\omega \rightarrow \omega/\omega_{pi}$, $V \rightarrow eV/T_e$) The upper (red) curves show the impedance for a *glancing angle resistive sheath* (**b**·**s** = 0.1 , ω = 0.1 ω_{pi}, Ω = 0.1 ω_{pi}) where **b** = **B**/B. The lower (blue) curves show the impedance curves for a *perpendicular capacitive sheath* (**b**·**s** = 1 , ω = 9 ω_{pi}, Ω = any). Note that |z| is a generally increasing function of V_{pp} and that z goes to a constant as $V_{pp} \rightarrow 0$, since in that limit the problem is strictly linear. The rich and complicated behavior of the impedance curves is discussed in some detail in Ref. [3]. For present purposes, our goal is to illustrate that the generalized sheath BC gives solutions with complex z, with the resistive power dissipation physics also included.

The effect of varying the rf sheath BC in the coupled micro-macro problem is illustrated in Fig. 2. The result labeled C uses the capacitive limit in the impedance formalism, Re(z) = 0. Note that model Z is not the complete impedance, because the present ad hoc model neglects the ion currents. In both models, there is a similar root structure with 3 roots (black, red and blue) and an S-shaped curve.

Future work will develop analytic and numerical fits for the impedance computed by the microscopic model and include them in the macroscopic model solutions.

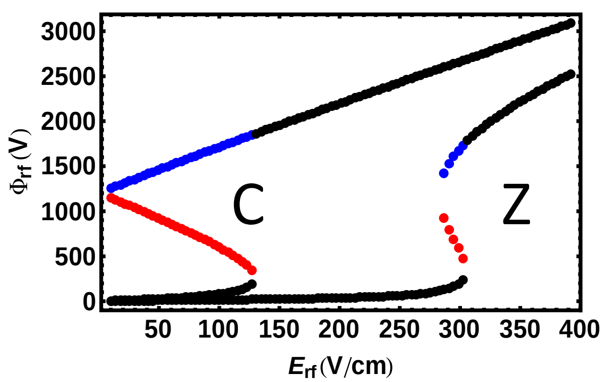

FIGURE 2. Macroscopic solutions for the rf sheath potential vs the amplitude of the incoming FW for different sheath BCs. The left-hand solution uses the capacitive (C) sheath BC (impedance formalism with Re z = 0); the right-hand solution (Z) is for the ad hoc micro-model (neglecting only the ion currents in the sheath). This model is an approximation to the full impedance BC.

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