Slow Wave Propagation and Sheath Interaction for ICRF Waves in the Tokamak Scrape-off-Layer

J.R. Myra and D.A. D'Ippolito
Lodestar Research Corp., Boulder, Colorado USA

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Motivation and background

- interested in ICRF sheath interaction with walls and limiters
  - sputtering, impurities, power loss, …
- rf sheaths generated primarily by the $E_{||}$ component ⇒ slow wave (SW)
- previously studied situations in which the FW can access the wall directly
  - poor central absorption or surface wave phenomena
  - FW generates SW to satisfy BC when wall normal has a component along B
- also have studied sheaths on the antenna structure
  - B-field tilt wrt. current strap ⇒ SW generation [D'Ippolito PoP 2009; & this meeting]
- now consider case where SW is generated at antenna and propagates/evanesces into SOL
  - this poster: model problems with simple rectangular geometry and constant density plasma ⇒ seek concepts and insight
  - in progress (H. Kohno et al.): numerical solution of SOL propagation with sheath BCs in a realistic SOL geometry ⇒ quantitative prediction
Geometry of Slow Wave (SW) excitation

- SW components, i.e. $E_{||}$, generated at antenna: e.g. protection tips and possibly at side-wall gaps (from B-field misalignment)
- SWs generated from localized source in $z$ and $x$
- understand their propagation & interaction with limiter sheaths
- Que: How much of the antenna sheath voltage appears across limiter sheaths; How much is dropped across the plasma?
SW is emitted through an aperture (source) into a box (SOL) bounded by conducting walls (limiters).

study the propagation/spreading/evanescence of the SW and its interaction with wall sheaths.

previous work (PRL 2008): the tenuous plasma limit $n_e < n_{lh} (\omega > \omega_{lh})$

- SW propagates as resonance cone (RC) without spreading
- reflects off of wall and generates self-consistent rf sheath

present work: the dense plasma limit $n_e > n_{lh} (\omega < \omega_{lh})$

- SW is normally evanescent, but here we will see it is not
Review: tenuous plasma limit: Resonance Cones (RCs)

- tenuous plasma model
  \[ \bar{\varepsilon} = (\bar{I} - bb) + bb\varepsilon_{||} \]

- EM SW dispersion relation
  \[ n_x^2 = -\varepsilon_{||}(n_z^2 - 1) \]

- ES limit
  \[ k_x = -(\omega_{pe} / \omega)k_z \]

- sheath BC (vacuum gap \( \Delta \))
  \[ E_x \mp \partial_x \Delta \varepsilon_{||} E_z = 0 \]

- use method of images to construct solution satisfying sheath BC

- key parameter is
  \[ \Lambda_{RC} = -\frac{\Delta \varepsilon_{||}}{a_{||}} \]
  \( a_{||} \) = parallel scale-length of RC
Tenuous plasma (cont’d):
RC Sheath voltage transmission for self-consistent $\Delta$
shows a threshold at $\Lambda_0 \sim 1 - 4$


- use Child-Langmuir law:
  make $\Delta$ consistent with fields at wall

\[ \Delta = \lambda_{de} \left| \frac{\alpha e V_{sh}}{T} \right|^{3/4} \]

- estimates for C-Mod show
  $\Lambda_0 \sim 4$ occurs when RC structures $\sim 200$ V are launched with parallel scale $a_{||} < 15$ cm.
High density SOL: model problem

- constant density plasma
- symmetric: consider modes even in $E_\parallel$
- local SW dispersion relation in plasma region $\varepsilon_\perp n_x^2 = \varepsilon_\parallel (\varepsilon_\perp - n_z^2)$
- sheath BC at wall, $z = L$ $E_x = -ik_x \Delta \varepsilon_\parallel E_z$
- determines a global dispersion relation $\Rightarrow$ eigenmodes of box

$$\eta \tan \eta = (\eta^2 + b^2)\Lambda$$

where

$$\eta = k_z L$$

$$b^2 = -\varepsilon_\perp \eta_0^2$$

$$\Lambda = -\frac{\Delta \varepsilon_\parallel}{L}$$

- procedure:
  - first find a complete set of eigenfunctions for the box that satisfy sheath BCs in $z$ and are outgoing/evanescent in $x$
  - expand the source (at $x = 0$) in this basis set
  - summed eigenfunction behavior determines solution at $x > 0$
Eigenfunctions of the box

\[ \eta \tan \eta = (\eta^2 + b^2)\Lambda \]

- \( \Lambda = 0 \) is metal wall limit \( \Rightarrow \cos(k_mz) \) with \( \eta_m = m\pi, \quad m = 0, 1, 2, \ldots \)
- \( \Lambda = \infty \) is insulating limit \( \Rightarrow \cos(k_mz) \) with \( \eta_m = m\pi/2, \quad m = 1, 3, 5 \ldots \)
- Intermediate \( \Lambda \) roots transition, but there is also a new root with pure imaginary \( \eta \Rightarrow \) sheath-plasma wave (SPW)

Roots for \( b = 0.1 \),
Re (blue),
Im (dashed magenta).
SPW can be localized to sheaths or global

• field pattern $\text{Re}[E_z(x,z)]$ for the SPW eigenmode (imaginary root)
  • $\Lambda \ll 1 \Rightarrow \text{Im}(\eta) > 1 \Rightarrow$ mode hugs the sheath boundary [see also D’Ippolito PoP 2006, Myra PRL 1991]
• projection of localized source onto SPW is small for $\Lambda \ll 1$ or $\Lambda >> 1$

Projection $C_\xi$ onto the SPW vs. $\Lambda$
  • recover metallic and insulating complete sets for $\Lambda \ll 1$ or $\Lambda >> 1$, without the SPW.
  • note SPW resonance near $\Lambda = 1$. 
Insights from the metal wall limit $\Lambda = 0$

$$E_z = \sum_m C_m \cos k_{mz} z \ e^{i k_{mx} x}$$

at $x = 0$: \[ \sum_m C_m \cos k_{mz} z = \delta(z) \]

- $b >> 1 \Rightarrow$ evanescence on the scale $x \sim \delta_e$
  - fields do not reach the wall at $z = \pm L$
  \[ E_z = \delta(z) \ e^{-x/\delta_e} \]

- $b << 1 \Rightarrow$ spreading in $z$ and evanescence in $x$
  \[ 2LE_z = e^{-x/\delta_e} - 2 + \frac{1}{1 - e^{i\pi(z/L + x/h)}} + \frac{1}{1 - e^{i\pi(-z/L + x/h)}} \]
  \[ h = b\delta_e \]
  - fields reach the wall
  - short scale structures in $z$ are ES and decay quickly in $x \sim h \sim (m_e/m_i)^{1/2} L$
    - could create hot electrons on radial scale $(m_e/m_i)^{1/2} v_e \sim \rho_s$
  - long scale structures in $z$ ($k_z = 0$) are EM and decay more slowly in $x \sim \delta_e$

- because the fields reach the wall for $b << 1$, they will generate an rf-sheath with finite $\Delta \propto \Lambda$
  - the limit $\Lambda = 0$ is not self-consistent $\Rightarrow$ need general $\Lambda$ expansion
Solution for fixed finite $\Lambda$

• Gaussian source (aperture) \( S(z) = \frac{1}{(2\pi)^{1/2}} \frac{a}{2a^2} \exp\left(-\frac{z^2}{2a^2}\right) \)

• \( b << 1 \) of most interest: so fields reach the wall \( \Rightarrow L < \delta_i \)

• for \( b << 1, \Lambda > 1 \), imaginary root approaches \( \eta = \text{i}b\left(\frac{\Lambda}{\Lambda - 1}\right)^{1/2} \)

• associate this root with the Alfvén mode from \( \Lambda \rightarrow \infty \) limit

\[
\eta^2 + b^2 = 0 \quad \Leftrightarrow \quad k_z v_a^2 = \frac{\omega^2}{1 - (\omega/\Omega_i)^2}
\]

• Alfvén resonance normally occurs for real \( k_z \) and \( \omega < \Omega_i \). Here, \( \omega > \Omega_i \) but imaginary \( k_z \) allowed by sheath BCs
Field pattern and emergence of SPW for specified $\Lambda$

- **left:** $|E_z(x, z)|$ for $b = 0.1$, $a = 0.1$ and specified value of $\Lambda = 3$.
- **right:** $\text{Re}[E_z(x, z)]$ for the same case.
- note the appearance of asymptotic fields in $x$
  - radially propagating mode = Alfvén sheath plasma wave (SPW)
  - mode follows the sheath boundary radially into the plasma
A substantial fraction of the source voltage ends up on the sheaths

- total V not conserved because of EM effects
- note that $V_{sh} \sim \text{const}$ for large $x$
Self-consistent solution for \( \Lambda, \Delta, \) and \( V_{sh} \) can have multiple roots

- use Child-Langmuir law: make \( \Delta \) consistent with fields at wall

\[
\Delta = \lambda_{de} \left[ \frac{\alpha e V_{sh}}{T} \right]^{3/4}
\]

\[
V_{sh} = \Delta \varepsilon_{||} E_z (z = L)
\]

comes from matching \( \varepsilon_{||} E_z \) across sheath-plasma interface

- graphical roots of \( V_{sh} + V_{th} = \left( \frac{\Lambda}{\Lambda_0} \right)^{4/3} = V_{CL} \)

\[
V_{CL} (\Lambda_0 = 0.34)
\]

\[
V_{sh} + V_{th}
\]

\[
\Lambda
\]

\[
b = 0.1
\]

multiple roots

\[
b = 0.8
\]

single root
Self-consistent sheath voltage (at $x \to \infty$) from SPW

- strong amplification possible for $b << 1$ in SPW resonant case ($\Lambda \sim 1$)
  - analogous effect seen in far-field “wave scattering” problem
    [D'Ippolito PoP 2008]
- as $b$ increases to $b \sim 1$, $V_{sh}$ decreases, resonant structure and multiple roots disappear
  - get critical $\Lambda_0$ at which sheath goes from thermal to rf-dominated
Summary

- SW fields emitted by a localized source propagate and evanesce into the SOL.
- SW interaction with the wall, and concomitant rf-sheath formation is possible in some parameter regimes:
  - Tenuous plasmas \( n_e < n_{lh} \) (\( \omega > \omega_{lh} \)) for which the SW propagates as resonance cone (RC) without spreading.
  - Dense plasmas \( n_e > n_{lh} \) (\( \omega < \omega_{lh} \)) with nearby limiters, typically \( L_\parallel < \delta_i \).
- In the dense plasma case studied here, the mechanism for wave-field coupling to the wall sheaths involves the sheath-plasma wave, for which we obtain a new electromagnetic dispersion relation related to the Alfvén wave.
- The SPW carries the SW fields and sheath voltages radially into the plasma along the surface of metal structures.
- A numerical treatment of this problem in realistic SOL geometry will be very interesting, and is in progress [H Kohno et al.]