Edge Power Dissipation and Cavity Q for ICRF Eigenmodes*

J.R. Myra and D.A. D'Ippolito

Lodestar Research Corp., Boulder, CO

*Work supported by U.S. DOE contract DE-FG02-88ER53263.

presented at the Thirty-Sixth Annual Meeting of the Division of Plasma Physics, Minneapolis, Minnesota, 7-11 November, 1994, poster 4R26.

Abstract

When the central absorption of ICRF wave power is weak, power dissipation at the edge must be examined as a possible competitive mechanism. In low single pass current drive experiments, the edge power absorption determines the effective number of single passes a ray experiences, and thus determines the minimum useful core single pass absorption fraction. In this paper, we survey a number of edge dissipation mechanisms including wall resistivity, Coulomb and neutral collisions, parametric decay, and dissipation by far-field sheaths. For the latter, a new analytical model of power balance in an rf sheath has been developed. An expression for the cavity Q of fast wave eigenmodes is obtained including these effects together with central absorption. Preliminary theoretical results indicate that while the edge processes can affect impurity sputtering and edge conditions, they do not result in substantial rf power dissipation except in very low single pass absorption cases. Numerical examples are given for low single pass current drive experiments on DIII-D.

for a quick tour of this poster, read the highlights marked by yellow in the left margin

Introduction & motivation

CD experiments on DIII-D in very low single pass absorption regimes show an anomalous loss of wave energy at the edge ~ 4 % per pass [R. Pinsker, et al., 3F9].

Some low single pass experiments on TFTR show evidence for edge interactions and impurity injection [K. Hill, et al., Poster 6Q14].

In low single pass cases, the edge power absorption determines the effective number of single passes a ray experiences, and thus determines the minimum useful core single pass absorption fraction.

In this paper we survey several edge dissipation mechanisms:

- Coulomb collisions
- Far field sheaths
- neutral collisions
- resistive walls
- other mechanisms: PDI, radiation from tokamak, ...

Numerical examples are given for DIII-D low single pass CD experiments

Relation between cavity Q, damping rate γ_{eff}, and single pass absorption fraction A_{sp}

$$A_{sp} = 2\gamma_{eff}\tau, \;\; \gamma_{eff} = \omega/2Q, \;\; \tau = 2a/v_A$$

$$A_{sp} = \omega\tau/Q = \pi\nu/Q \;(\nu = \text{radial mode number})$$

For weak dissipation ($A_{sp} << 1$)

$$1/Q = \Sigma_j \,(1/Q_j)$$

$$\text{power loss per process} = P_j = PQ/Q_j$$

note: Dissipation in the FW eigenmode problem (here) is different from anomalous power loss from the antenna. The latter would include near field sheath dissipation (ions falling down sheath), Fermi acceleration of electrons, and parasitically launched modes (SPW, surface waves).

Coulomb collisions

Ion-electron collisions

General theory

The ion kinetic equation with i-e collisions is

$$\frac{\partial f}{\partial t} = \ldots C_{ie} = \ldots - \frac{4\pi Z^2 e^4 \ln\Lambda}{m_e m_i} \nabla_v \cdot \left(f \int d^3v' \, \frac{f_e(\mathbf{v}') \, (\mathbf{v}'+\mathbf{U}_e-\mathbf{U}_i)}{|\mathbf{v}'+\mathbf{U}_e-\mathbf{U}_i|} \right)$$

where $\mathbf{U}_e$ and $\mathbf{U}_i$ are the electron and ion drift velocities.
Taking the $\mathbf{v}$ moment leads to the an ion momentum equation of the form

$$\frac{\partial \mathbf{U}_i}{\partial t} = \ldots - \nu_i \, (\mathbf{U}_i - \mathbf{U}_e) \text{ with } \nu_i \equiv \nu_{ie} = \frac{4(2\pi)^{1/2} n_e Z^2 e^4 \ln\Lambda m_e^{1/2}}{3 \, m_i T_e^{3/2}}$$

By conservation of total i,e momentum the electron equation must have the form

$$\frac{\partial \mathbf{U}_e}{\partial t} = \ldots - \nu_e \, (\mathbf{U}_e - \mathbf{U}_i) \quad \text{with } \nu_e \equiv \nu_{ei} = \frac{m_i}{m_e} \nu_i$$

We determine the dielectric tensor including collisions by inverting

$$\begin{pmatrix} -i\omega+\nu_i & -\Omega_i & -\nu_i & 0 \\ \Omega_i & -i\omega+\nu_i & 0 & -\nu_i \\ -\nu_e & 0 & -i\omega+\nu_e & \Omega_e \\ 0 & -\nu_e & -\Omega_e & -i\omega+\nu_e \end{pmatrix} \begin{pmatrix} U_{ix} \\ U_{iy} \\ U_{ex} \\ U_{ey} \end{pmatrix} = \begin{pmatrix} eE_x\_m_i \\ eE_y\_m_i \\ eE_x\_m_e \\ eE_y\_m_e \end{pmatrix}$$

and then forming $\mathbf{J} = en_e(\mathbf{U}_i - \mathbf{U}_e) = \sigma\cdot\mathbf{E}$, and finally $\varepsilon = 4\pi i\sigma/\omega$. To leading order in collisionality and mass ratio, the components of ε relevant to FW propagation are

$$\varepsilon_\perp = 1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2-\Omega_i^2} + \frac{i\nu_i \, \omega_{pi}^2 \omega(\omega^2+\Omega_i^2)}{\Omega_i^2(\omega^2-\Omega_i^2)^2}$$

$$\varepsilon_x = \frac{\omega\omega_{pi}^2}{\Omega_i(\omega^2-\Omega_i^2)} - \frac{2i\nu_i \, \omega_{pi}^2\omega^2}{\Omega_i(\omega^2-\Omega_i^2)^2}$$

Note: In the low freq limit, $\omega \ll \Omega_i$, collisional effects vanish from ε_x because electron and ions E×B drift together.

Quick derivation of the high frequency limit $\omega >> \Omega_i$

FW dispersion relation for $\omega >> \Omega_i$ is

$$n_\perp^2 = -\frac{\varepsilon_x^2}{\varepsilon_\perp} \approx \frac{\omega_{pi}^2}{\Omega_i^2} \quad \Rightarrow \quad \omega \approx k_\perp v_A$$

To add collisions, big term is $\nu_i \mathbf{U}_e$ in the ion momentum equation:

$$\frac{\partial \mathbf{U}_i}{\partial t} = \dots - \nu_i (\mathbf{U}_i - \mathbf{U}_e)$$

since in the high frequency limit $U_e \sim cE/B$, $U_i \sim ZeE/m\omega$, $U_e/U_i \sim \omega/\Omega_i >> 1$.

Thus

$$\frac{\partial \mathbf{U}_i}{\partial t} = \frac{Ze}{m}\left(\mathbf{E} \quad + \quad \frac{\nu_i}{\Omega_i} \mathbf{E} \times \mathbf{e}_z\right)$$

$\Rightarrow$ conductivity is modified: $\sigma \rightarrow \sigma + \dfrac{\nu_i}{\Omega_i} \sigma \times \mathbf{e}_z$

Dominant effect is that

$$\varepsilon_\perp \rightarrow \varepsilon_\perp + \frac{i\nu_i}{\Omega_i} \varepsilon_x = -\frac{\omega_{pi}^2}{\Omega_i^2} + \frac{i\nu_i \omega_{pi}^2}{\omega \Omega_i^2}$$

FW dispersion relation is modified:

$$\delta n_\perp^2 = n_{\perp 0}^2 \frac{\delta \varepsilon_\perp}{\varepsilon_\perp} \Rightarrow \delta\omega = \frac{i\nu_i \omega^2}{2\Omega_i^2}$$

Ion-ion collisions

For a rough estimate, we take the local damping rate to be

$$\gamma \sim \nu_{ii} \, (k_\perp \rho_i)^2$$

where

ν_{ii} = ion - ion collision frequency

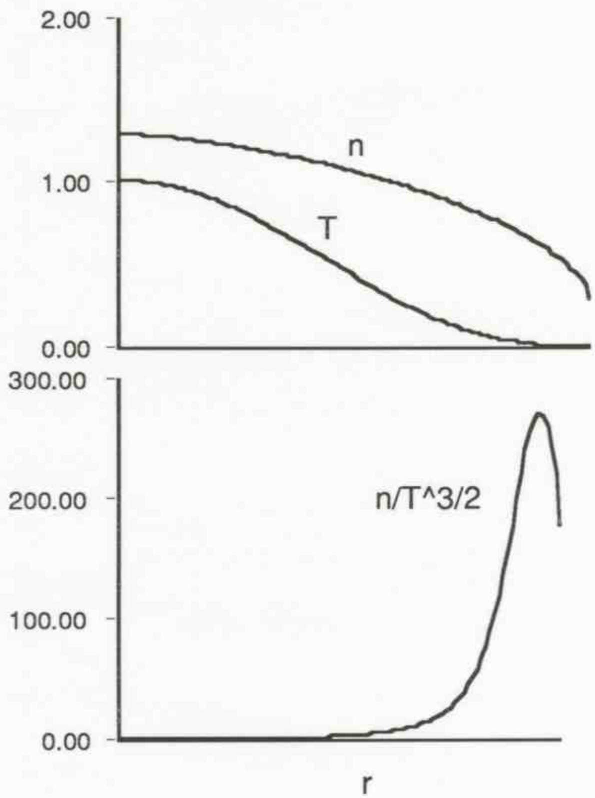
$k_\perp \sim \omega / v_A$

Estimating the local and global damping rates

Typically, collisions are largest at the edge, so we use edge parameters to estimate the local damping rates for the collisional processes. Then the Q for the given process is given by

$$Q = \frac{\omega V}{2\gamma V_{edge}}$$

where $V_{edge}/V = 2\Delta r/a$ is the volume where collisions are significant. Typically, $\Delta r/a \sim 0.15$ as the following figure shows.



Numerical estimates of Coulomb dissipation

Inputs

f	60	MHz	wave freq
ln lambda	15		Coulomb logarithm
ne	6e12	cm^-3	density
Te	25	eV	electron temperature
Ti	50	eV	ion temperature
mu	2	amu	ion mass
B	10.9	kG	magnetic field
coll frac	0.15		fraction of minor radius over which collisionality is operative
Zeff	4.3		effective Z
a	55	cm	minor radius

Outputs

w	3.76e8	rad/s	wave freq
nu e-i	8.97e6	s^-1	electron collision rate with ions
nu i-e	2.44e3	s^-1	ion collision rate with electrons (slowing down rate)
gamma i-e	6.37e4	rad/s	local damping rate due to i-e and e-i collisions
wci	5.22e7	rad/s	ion cyclotron freq
w/wci	7.22		wave freq / ion cycl freq
wpi	2.28e9	rad/s	ion plasma freq
k perp	0.55	cm^-1	FW wavenumber for k par = 0
rho i	0.093	cm	ion gyroradius
nu i-i	8640	s^-1	ion collision rate with ions
kperp*rho	0.051		FLR parameter
gamma i-i	22.9	rad/s	local damping rate due to nu i-i with FLR
corr factors	0.3		volume ratio
Q i-e	9.85e3		Cavity Quality factor due to gamma i-e
Q i-i	2.74e7		Cavity Quality factor due to gamma i-i
gamma eff	1.91e4	rad/s	total gamma times corr factors
gamma/w	5.07e-5		gamma eff / w
tau sp	1.60e-7	s	time for wave to make a single pass
Asp	0.6	%	energy loss per single pass
nu	19		w tau / pi = radial mode number

Summary: Coulomb collisions

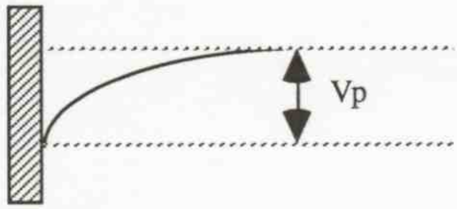
i-e collisions are dominant

Q ~ 9.8e3

A_{sp} ~ 0.6 %

Far field sheaths

Simple model of rf sheath rectification



plasma potential relative to wall is

$$V_p = V_{dc} + V_0 \cos(\omega t)$$

Integrating over Maxwellian, including that portion of tail with $mv^2/2 > |eV_p|$

$$J_e = -n_0 e v_e (2\pi)^{-1/2} \exp(-eV_p/T_e)$$

$$J_i = n_0 e u; \;\; \text{where } u \approx c_s \text{ (immobile ion limit)}$$

Averaging over time

$$\langle J_e \rangle = -n_0 e v_e (2\pi)^{-1/2} \exp(-eV_{dc}/T_e)\, I_0(eV_0/T_e)$$

Setting $\langle J \rangle = 0$ gives the rectified (dc) voltage:

$$\frac{eV_{dc}}{T_e} = 3 + \ln\left[I_0\left(\frac{eV_0}{T_e}\right)\right] \quad \text{where "3"} \Rightarrow \ln\left(\frac{v_e}{u(2\pi)^{1/2}}\right) \approx \frac{1}{2}\ln\left(\frac{m_i}{2\pi m_e}\right)$$

Aside: Note that for $eV_0 >> T_e$, $eV_{dc} \sim 3T_e + eV_0 - (T_e/2)\ln(2\pi eV_0/T_e)$

Sheath energy transmission factor for electrons

Using the same truncated Maxwellian model, calculate the electron particle and heat fluxes

$$\Gamma_e \equiv \int_{vp} dv_{||}\, v_{||}\, f_m = n_0 v_e (2\pi)^{-1/2} \exp(-eV_p/T_e)$$

$$H_e \equiv \int_{vp} dv_{||}\, v_{||}\, (mv^2/2)\, f_m = \Gamma_e\,(2T_e + eV_p)$$

hence the electron sheath energy transmission factor is

$$\gamma_e \equiv H_e/\Gamma_e T_e = 2 + eV_p/T_e \ (\rightarrow 5 \text{ for the usual Bohm sheath}).$$

Note that this H_e is the kinetic energy flux lost from the plasma. The kinetic energy flux observed at the wall would be smaller since γ_e would not have the $3T_e$ term. (Agrees with Stangeby in Plasma Wall Interactions, Sec. 5, pp. 54 - 58)

Now consider the rf case

$$\langle\Gamma_e\rangle = n_0 v_e (2\pi)^{-1/2} \exp(-eV_{dc}/T_e)\, I_0(eV_0/T_e)$$

$$\langle H_e\rangle = n_0 T_e v_e (2\pi)^{-1/2} \exp(-eV_{dc}/T_e)\, [(2 + eV_{dc}/T_e) I_0(z) - I_0'(z)]; \quad z = eV_0/T_e$$

$$\gamma_e \equiv \langle H_e\rangle/\langle\Gamma_e\rangle T_e = 2 + eV_{dc}/T_e - zI_1(z)/I_0(z)$$

or using the result for V_{dc}

$$\gamma_e = 5 + \ln[I_0(z)] - zI_1(z)/I_0(z)$$

Simple model of power balance in an rf sheath

Capacitor plate sheath model, with voltages at left and right plates

$$V_1 = V_0 \cos(\omega t) - V_{dc}; \quad V_2 = -V_0 \cos(\omega t) - V_{dc}$$

Total current flowing through the device is

$$I_{thro} = \frac{A}{2} \frac{n_0 e v_e}{(2\pi)^{1/2}} [\exp(eV_1/T_e) - \exp(eV_2/T_e)]$$

Thus the dissipated power is

$$P \equiv \langle (V_1 - V_2) I_{thro}\rangle = 2(2\pi)^{-1/2} A n_0 e v_e V_0 \exp(-eV_{dc}/T_e)\ I_0'(z)$$

Combining the above expressions we can construct an energy balance relation

$$\langle H\rangle + \langle P\rangle/A = \langle\Gamma\rangle T_e (2 + eV_{dc}/T_e)$$

where the terms are

$\langle H\rangle$ = heat flux from presheath

$\langle P\rangle/A$ = rf power dissipated in sheath $\Rightarrow$ will be used for $Q_{far\text{-}field}$

$2\langle\Gamma\rangle T_e$ = electron thermal energy at wall

$\langle\Gamma\rangle eV_{dc}$ = ion energy at wall after falling through voltage V_{dc}

Far field sheath dissipation

Unabsorbed FW energy is converted to SW ⇒ sheaths at the walls. The conversion depends on the degree of mismatch between the wall and flux surface.

[Perkins, (1989); Myra, D'Ippolito and Bures, Phys. of Plasmas 1, 2890 (1994)]

$eV_0/T_e < 1$ usually relevant for far field sheaths (good to 30% even if $eV_0/T_e \sim 2$)

$$P = \frac{A_\perp n_0 c_s (eV_0)^2}{4T_e}$$

$A_\perp$ = projected area of all sheaths = $2\pi a \lambda_n f_{rf}$

f_{rf} = fraction of sheaths covered by rf

λ_n = density gradient scale length

V_0 = (0 - p) rf driving voltage

Need to estimate V_0 in terms of $B_z \equiv \psi$ at the edge

$$V_0 = 2E_{||}L_{||}$$

$$E_{||} = E_\perp(\varepsilon_\perp/\varepsilon_{||})^{1/2} \sim E_\perp(m_e/m_i)^{1/2}$$

$$E_\perp = (\omega/c)\,\psi h$$

h = radial scale length of flux surface-boundary mismatch

$L_{||} = 1/k_{||sw} \sim (m_i/m_e)^{1/2}\, w/\pi$, w = poloidal scale of flux surface-boundary mismatch

$V_0 = (2/\pi c)\, w\omega\psi h$

Thus the power dissipated in far field sheaths is

$$P = \frac{A_\perp n_0 c_s}{T_e}\left(\frac{e}{\pi c} w\omega h\right)^2 \langle|\psi|^2\rangle_A$$ where $\langle ... \rangle_A$ implies a surface average

The stored energy in the FW scales as

$W = (\pi/4)a^2R \langle|\psi|^2\rangle_V$ where $\langle ... \rangle_V$ implies a volume average

Using Q = ωW/P we obtain

$$Q = \frac{\pi^2 a R c^2 T_e R_\psi}{8 n_e c_s e^2 w^2 \omega h^2 \lambda_n f_{rf}}$$

where $R_\psi = \langle|\psi|^2\rangle_V / \langle|\psi|^2\rangle_A \sim 1$ if there is no edge evanescence

Numerical estimates of far field sheath dissipation

Inputs

f	60	MHz	wave freq
nea	6e12	cm^-3	density (edge)
Te	25	eV	electron temperature
mu	2	amu	ion mass
a	55	cm	minor radius
R	179	cm	major radius
lambda n	2	cm	edge density grad scale length
R-psi	1		vol_avg/area_avg psi (= 1 if no evanescence)
gamma Z	9		factor that occurs in cs
ww	1	cm	poloidal scale of flux surface to wall mismatch
h	4	cm	radial scale of flux surface to wall mismatch
f_rf	1		fraction of sheath Aperp that is covered by rf
B	10.9	kG	magnetic field
Inputs for alternate derivation			
P_Bohm	1	MW	total power into Bohm sheaths
gamma_e	5		electron sheath energy transmission factor

Outputs

Qff	2618		cavity Q factor due to FF sheaths
Qff alternate	2115		alternate derivation in terms of P_Bohm
w	3.77e8		
gamma eff	7.19e4	s^-1	w /(2 Qff)
va	6.85e8	cm/s	Alfven velocity
tau sp	1.60e-7	s	time for wave to make a single pass
Asp	2.3	%	energy loss per single pass

Summary: far field sheaths

$Q \sim 2.6e3$

$A_{sp} \sim 2.3$ %

Ion & electron collisions with neutrals

The Q from neutral collisions is given by

$$Q = \frac{a}{2\lambda_0} \frac{\omega}{2\gamma}$$

$a/2\lambda_0$ = volume ratio of plasma containing neutrals

λ_0 = neutral penetration length (radially)

γ = local damping rate due to ion-neutral collisions

Ion-neutral collisions (CX)

Unlike the case of e-i collisions which must conserve momentum of electrons + ions, we expect the ion-neutral collisions to enter the momentum equation as

$$\frac{\partial \mathbf{U}_i}{\partial t} = \ldots - \nu_{i0} \mathbf{U}_i \quad \text{where } \nu_{i0} = n_0 <\sigma v_i> \text{ and } n_0 = \text{neutral density}$$

$\Rightarrow$ replacement $\omega \rightarrow \omega + i\nu_{i0}$ in ion $\sigma = \omega\varepsilon/4\pi i$

High freq limit ($\omega >> \Omega_i$):

$$\varepsilon_\perp \rightarrow -\omega_{pi}^2/\omega(\omega+i\nu_{i0}), \quad \varepsilon_x = \omega_{pi}^2/\omega\Omega_i$$

$$n_\perp^2 = -\frac{\varepsilon_x^2}{\varepsilon_\perp} \approx \frac{\omega_{pi}^2(\omega+i\nu_{i0})}{\Omega_i^2\omega}$$

and the local damping rate is

$$\gamma_{cx} = -\nu_{i0}/2$$

This gives $Q = a\omega/2\lambda_0\nu_{i0}$, so next eliminate λ_0 in terms of the CX cross section:

$$\lambda_0 = \alpha v_0/\nu_{0i} \text{ where } \nu_{0i} = n_i <\sigma|v_0 - v_i|> = n_e <\sigma v_i>$$

$\alpha \sim 1$ is a numerical factor, v_0 = neutral velocity

This gives Q in terms of $\nu_{0i}/\nu_{i0} = n_e/n_0$

Determine neutral to edge electron density ratio from particle balance assuming perfect recycling

$$n_e c_s (2\pi a)\lambda_n = n_0 v_0 (2\pi R)(2\pi a)$$

$$\frac{n_0}{n_e} = \frac{c_s \lambda_n}{2\pi R v_0}$$

Thus

$$Q_{cx} = \frac{\pi R a \omega}{\alpha c_s \lambda_n}$$

Independent of v_0 and the cross-section!

Electron-neutral collisions (ionization, excitation)

Now have $\omega \rightarrow \omega + i\nu_{e0}$ in electron $\sigma = \omega\varepsilon/4\pi i$

Adding electron and ion contributions, retaining the small collision terms yields

$$\varepsilon_x = \frac{\omega_{pi}^2}{\omega \Omega_i}\left(1 + \frac{2i\nu_{e0}\omega}{\Omega_e^2}\right)$$

$$\varepsilon_\perp = -\frac{\omega_{pi}^2}{\omega^2}\left(1 - \frac{i\nu_{e0}\omega}{\Omega_e \Omega_i}\right)$$

and the local damping rate takes the form

$$\gamma_{e0} = -\frac{\omega^2 \nu_{e0}}{2\Omega_e \Omega_i}$$

Comparison with CX:

Typical CX rate is $<\sigma v_i> = 2e\text{-}8\ cm^2/s$

Typical electron-neutral collision rate is $<\sigma v_e> = 2e\text{-}8\ cm^2/s$ also
(even though $v_e >> v_i$)

So $\nu_{e0} \sim \nu_{i0}$ and $\gamma_{e0} << \gamma_{cx}$ because of the (ω/Ω_e) factor.

Electron-neutral collisions lead to negligible dissipation.

Numerical estimates of ion-neutral dissipation

Inputs

f	60	MHz	wave freq
nea	6e12	cm^-3	density (edge)
Te	25	eV	electron temperature
Ti	50	eV	ion temperature
mu	2	amu	ion mass
a	55	cm	minor radius
R	179	cm	major radius
<sig vi>	2e-8	cm^3/s	CX cross section
lambda n	2	cm	edge density grad scale length
To	1	eV	neutral temperature
alpha	1		number ion CX mfp's the neutrals penetrate
B	10.9	kG	magnetic field

Outputs

w	3.76e8	rad/s	wave freq
nuio	1.50e3	s^-1	ion neutral collision freq (CX)
gamma	754.	rad/s	local damping rate
cs	4.89e6	cm/s	sound speed
vo	6.92e5	cm/s	neutral speed
no	7.54e10	cm^-3	neutral density
Q	1.19e6		cavity Q due to ion neutral collisions
gamma eff	158	rad/s	gamma * volume factor
va	6.85e8	cm/s	Alfven velocity
tau sp	1.60e-7	s	time for wave to make a single pass
Asp	0.01	%	energy loss per single pass

Summary: neutral collisions

ion-neutral collisions dominate over electron-neutral collisions, but even the ion collisions give negligible dissipation

$Q \sim 1.2e6$

$A_{sp} \sim 0.01$ %

Resistive wall

Classical theory

The power loss per unit area on the surface of a lossy conductor is [Jackson, "Classical Electrodynamics"]

$$\frac{dP}{dA} = \frac{\omega\delta}{16\pi}|\psi|^2$$

$\psi \equiv B_z \approx B_{tangential}$
δ = skin depth = $c/(2\pi\omega\sigma)^{1/2}$; σ = conductivity
The surface area of the torus covered by the lossy conductor is taken to be
$A = (2\pi R)(2\pi a)f_A$
f_A = fraction of total surface of torus covered by the lossy conductor

The stored energy in a FW eigenmode is approximately

$$W = (2\pi R)\pi a^2 \frac{\langle|\psi|^2\rangle_V}{8\pi}$$

Thus the Q is given by

$$Q = \frac{aR_\psi}{f_A\delta}$$

$R_\psi = \langle|\psi|^2\rangle_V / \langle|\psi|^2\rangle_A \sim 1$ if there is no edge evanescence

The above handwaving estimate is actually rigorous if the factor R_ψ is calculated from a FW eigenmode model: e.g. plasma cylinder model

i) Litwin and Hershkowitz, Phys. Fluids 30, 1323 (1987).
ii) Appert, Vaclavik, and Villard, Phys. Fluids 27, 432 (1984).
iii) Myra, D'Ippolito and Francis, Phys. Fluids 30, 148 (1987).

For example, for the simple case of no vacuum region, the Litwin model gives $\psi = J_m(k_f r)$ and

$$R_\psi = \frac{2}{x_a^2 J_m^2(x_a)} \int_0^{x_a} dx\, x\, J_m^2(x) \quad \text{with the BC that} \quad \frac{x_a J_m'(x_a)}{J_m(x_a)} = \frac{m\varepsilon_x}{\varepsilon_\perp}$$

For $\varepsilon_x/\varepsilon_\perp = 1$ it can be shown that $R_\psi = 1$ for all radial modes and all m. For $\varepsilon_x/\varepsilon_\perp \sim 1$ it turns out numerically that $R_\psi \sim 1$.

Numerical estimates of resistive wall dissipation

$$\delta(\text{cm}) = \frac{50.4}{[f(\text{MHz})\ \sigma(\text{siemens/m})]^{1/2}}$$

	σ(siemens/m)	δ(cm) at 50 MHz
Cu-alloy	2×10^{7}	1.6×10^{-3}
C (graphite)	1×10^{5}	2.3×10^{-2}
Be	1×10^{7}	2.3×10^{-3}

Inputs

f	60	MHz	wave freq
a	55	cm	minor radius
R-psi	1		vol_avg/area_avg psi (= 1 if no evanescence)
f_A	0.1		fraction of surface with resistive material
sigma	1.0e5	S/m	electrical conductivity
B	10.9	kG	magnetic field
nea	6e12	cm^-3	density (edge)
mu	2	amu	ion mass

Outputs

delta	2.05e-2	cm	skin depth in resistive conductor
Q	2.67e4		cavity Q
w	3.76e8		
gamma eff	7.05e3	s^-1	w /(2 Q)
va	6.85e8	cm/s	Alfven velocity
tau sp	1.60e-7	s	time for wave to make a single pass
Asp	0.2	%	energy loss per single pass

Summary: resistive wall

$Q \sim 2.7e4$

$A_{sp} \sim 0.2$ %

Parametric Decay Instability (PDI)

[Porkolab, Fusion Eng. Design 12, 93 (1990)]

Thresholds

Decay instability thresholds can be met in front of the antenna, but probably will not be met for waves reflecting off of the edge plasma, and thus will not impact Q or A_{sp}. For example, using

$$E_\perp = (64\pi\omega k_x R_A T_{sp} S)^{1/2}\, x\alpha/c$$

R_A = antennas / torus area — T_{sp} = single pass transmission

S = launched Poynting flux — x = distance from wall

α = tunneling factor (= 1 if no wave evanescence)

f = 60 MHz, n_{ea} = 6e12 cm^3, x = 4 cm, R_A = 0.025, $T_{sp} = \alpha = 1$, S = 0.3 kW/cm^2

$\Rightarrow$ $E_\perp \sim 70$ V/cm

Typical thresholds [Porkolab] are $E_{\perp thresh}$ = a few 100 V/cm

A_{sp} from energy loss of pump

Suppose that it were possible to exceed the threshold under some conditions. Consider PDI process FW(ω) $\rightarrow$ IBW(ω') + QM(ω''), where $\omega \sim \omega'$. Then energy loss rate of FW = energy gain of IBW + QM ~ energy gain of IBW, so

$$\gamma W = \gamma' W'$$

where $W \sim (\omega_{pi}^2/\Omega_i^2)\, |E|^2$ is the wave energy density. The forms of W and W' are different, but the proportionality to is $|E|^2$ of the same order, so

$$\gamma = \gamma' \frac{|E'|^2}{|E|^2}$$

For the FW, the single pass damping decrement is

$$A_{sp} = \gamma L/v_g, \quad \text{where L = length of the unstable region}$$

For the IBW, the growth from fluctuation level E_0' is given by

$$E' = E_0' \exp(A), \quad \text{where } A = \gamma' L/v_g'$$

Combining eqs yields

$$A_{sp} = \frac{v_g' \, |E_0'|^2}{v_g \, |E|^2} \; A \exp(2A)$$

which can be used for a direct estimate. Alternatively, noting that $A_{sp} \sim 1$ corresponds to pump depletion

$$A_{sp} \sim \frac{A}{A_{pd}} \exp[2(A - A_{pd})]$$

so for $A << A_{pd}$, A_{sp} is *exponentially* small.

Numerical estimates of PDI dissipation

E field estimate for threshold comparisons:

Inputs

f	60	MHz	wave freq
nea	6e12	cm^-3	density (edge)
mu	2	amu	ion mass
x	4	cm	radial distance from wall
B	10.9	kG	magnetic field
R_A	0.025		antenna area / surface area of torus
Tsp	1		single pass transmission coef
S	0.3	kW/cm^2	launched Poynting flux
alpha	1		tunelling factor

Outputs

w	3.76e8	rad/s	wave freq
wpi	2.28e9	rad/s	ion plasma freq
wci	5.22e7	rad/s	ion cyclotron freq
k perp	0.55	cm^-1	FW wavenumber for k par = 0
E perp	72	V/cm	

Asp for the PDI:

Inputs

Ti	100	eV	ion temperature (edge)
Eo'	10	V/cm	IBW fluctuation amplitude
E	72	V/cm	FW pump amplitude
L	10	cm	length of unstable region
gamma	1e6	s^-1	PDI growth rate

Outputs

vi	6.92e6	cm/s	ion thermal velocity
va	6.85e8	cm/s	Alfven velocity
A	1.44		PDI growth factor
Asp	0.5	%	FW damping decrement per pass

Results are very sensitive to E_0': $E_0' = 1$ V/cm $\Rightarrow A_{sp} = 0.005$ %

$E_0' = 100$ V/cm $\Rightarrow A_{sp} = 50$ %

Other mechanisms

core absorption

The edge processes must be compared against core processes.

$$A_{sp} = \omega\tau/Q = \pi\nu/Q \ (\nu = \text{radial mode number})$$

For very low single pass absorption current drive scenarios $A_{sp} \sim 0.05$. For high frequency current drive scenarios ($\omega >> \Omega_i$) ν can be large (e.g. in DIII-D $\nu \sim 15$). Thus, Q due to core absorption can be as high as $Q \sim 1000$.

PDI (Parametric Decay Instability)

[Porkolab, Fusion Eng. Design 12, 93 (1990)]

Decay instability thresholds can be met in front of the antenna, but probably will not be met for waves reflecting off of the edge plasma, and thus will not impact Q or A_{sp}. For example, using

$$E_{\perp} = (64\pi\omega k_x R_A T_{sp} S)^{1/2}\ x\alpha/c$$

R_A = antennas / torus area
T_{sp} = single pass transmission
S = launched Poynting flux
x = distance from wall
α = tunneling factor (= 1 if no wave evanescence)

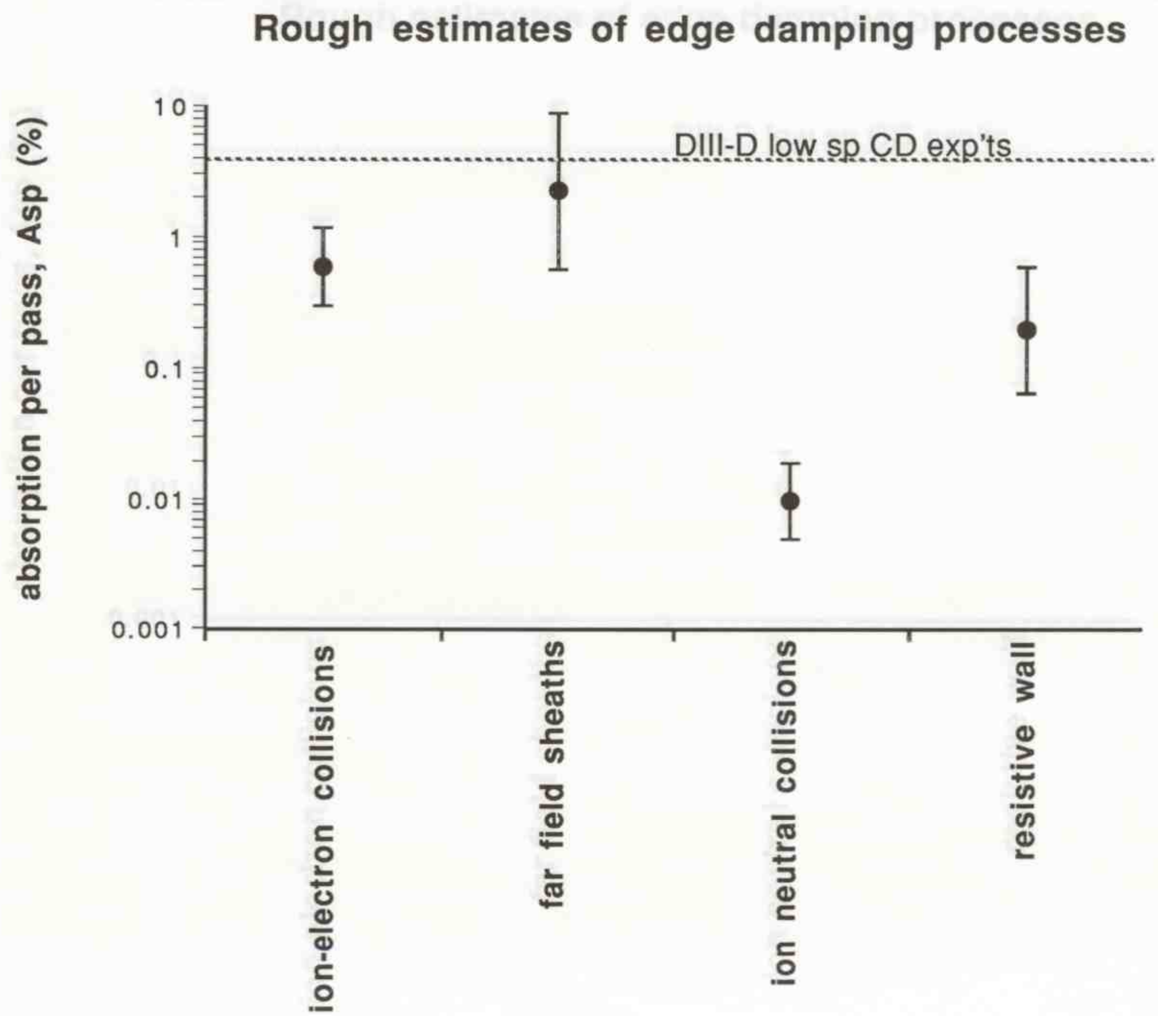
f = 60 MHz, n_{ea} = 6e12 cm^3, x = 4 cm, $R_A = 0.025$, $T_{sp} = \alpha = 1$, S = 0.3 kW/cm^2
$\Rightarrow E_{\perp} \sim 70$ V/cm
Typical thresholds [Porkolab] are $E_{\perp thresh}$ = a few 100 V/cm

radiation from tokamak

FW energy can leak out of the ports. A theoretical estimate of the power loss has not yet been attempted. Vacuum cavity Q measurement (Pinsker, private communication) could possibly help to assess this mechanism.

add your own here (we'll try to look at it)

Summary and conclusions



Of the edge dissipation mechanisms that have been examined to date, the dominant ones are

ion-electron collisions

far field sheaths

wall resistivity

Of these, it appears that far-field sheaths may be the biggest contributor to the edge damping observed (inferred) from the DIII-D CD experiments in the low single pass regime, but the uncertainty of the far-field sheath estimates is also largest. The best estimates to date predict a single pass edge damping of a few percent for the far-field sheath mechanism.

For the case of i-e collisions, we have derived the collisional modifications to the conductivity tensor, and shown that they are not obtained by setting $\omega \rightarrow \omega + i\nu_{ie}$. In the high frequency limit for fast waves ($\omega >> \Omega_i$) we obtain an enhanced collisional damping rate $\gamma = -\nu_{ie}\omega^2/2\Omega_i^2$.

Except in very low core absorption cases, it appears that none of the above mechanisms should seriously inhibit multiple wave passes in most tokamaks.